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Research Article

ANALYZING THE APPLICABILITY OF THE ARBITRAGE PRICING MODEL IN THE BRAZILIAN STOCK MARKET: A MACRO-ECONOMETRIC PERSPECTIVE

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ABSTRACT

This research study investigates the suitability and relevance of the Arbitrage Pricing Model (APM) in the context of the Brazilian stock market, represented by the B3 S.A (formerly BOVESPA) index. The objective is to unravel the intricate relationship between the stock market performance and various macroeconomic variables, including inflation, interest rates, exchange rates, industrial production, and Gross Domestic Product (GDP).The research employs a multifaceted approach, encompassing stationarity tests, co integration analysis, Granger causality tests, and Vector Error Correction Modelling(VECM) techniques. Initial stationarity tests confirm the need to work with first differences of the variables to attain stationarity, paving the way for cointegration analysis. The Johansen Cointegration Test reveals the presence of a long-term equilibrium relationship between the stock market index and the selected macroeconomic indicators. Intriguingly, the Granger causality tests unveil bidirectional causal relationships among these variables, indicating mutual influence both in the short term and long term. Moreover, the VECM analysis elucidates the interplay between GDP and exchange rates, emphasizing their significant impact on the stock market index over time. The model includes a negative Error Correction Term (ECT), that proves its return to equilibrium in the long run after short term changes.

Keywords: Arbitrage Pricing Model (APM), Brazilian Stock Market, B3 S.A. (BOVESPA) Index, Macroeconomic Variables, Cointegration Analysis, Vector Error Correction Model (VECM).

INTRODUCTION

The Arbitrage Pricing Model (APM) is one of the theories developed to measure the effectiveness of investment decision. Stephen Ross initialized APM (1976) as an alternative to the Capital Asset Pricing Model (CAPM). APM is a linear function modeling that attempts to explain the returns on financial assets by a series of factors, taking into account systematic risks. The goal of both models is to determine the expected rate of return of an asset. In the Arbitrage Pricing Theory (APT), it is accepted that the return on a security is formed by the factors in the sector and the market, and that there is a positive correlation between return and risk. These factors are variables such as gross national product, inflation, interest rate, exchange rate etc. As the number of securities increases, the non-systematic risk will decrease, but the systematic risk will not change. The return of a security can be explained as the sum of the risks carried by the security by considering the risk-free interest rate and variable factors.

The definition of APT with formulas and its identification with the factors used in the theory are two separate issues. This is because the theory for a particular stock or asset cannot fully explain the different factors to an investor. In practice and in theory, a stock may show different sensitivity to various factors. For example, the stock price of a successful firm in the energy sector may be very sensitive to crude oil and natural gas prices, while the stock of a personal care firm may be relatively less sensitive to the price of oil.

APT has left the assessment of the factors that may be effective for a particular stock to the investor or analyst. Some of the difficulties that investors may face in determining these factors are:

- The determination of each of the factors affecting a particular stock,
- Determining the expected returns for each of these factors,
- Determining the sensitivity of stocks to each of these factors.

RESEARCH PROBLEM

Identifying and quantifying each of the factors affecting a stock is no trivial matter, and is one of the reasons CAPM remains the dominant theory to describe the relationship between a stock's risk and return. Upon the numerous advances and advantages of APT over CAPM, studies on this theory are few as compared to those on CAPM. This is partially due the complexity of the theory; most times, APT appears to be difficult and takes time to be analyzed by investors. Consequently, this study attempts to analyze APT in a simplified yet profound manner to better understand the theory and explore the relationship that exists between stock and several other macroeconomic factors.

OBJECTIVES OF THE STUDY

This study tries to bring a new approach to APT testing by addressing several macroeconomic factors. Using a Vector Auto Regressive (VAR) model, the objectives of the study are as follow:

1. To determine whether APT can be tested in the economies of developing countries.
2. Exploring the extent to which macroeconomic factors affect securities returns in emerging economies.
3. Identify the nature of the relationship between the macroeconomic factor and stock returns in developing countries.

SIGNIFICANCE OF THE STUDY

This study can be a contributing factor to extend the pool of research on APT for a large number of financial practitioners and researchers; moreover, it can promote to building a general understanding and awareness of the key features of APT. The specific significance of the study is as follows:

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1. Determining whether APT is valid in the economy of developing countries,
2. Investigating the extent of the effect of macroeconomic factors on stock yields in emerging economies,
3. Determining and defining the relationship between macroeconomic factors and stock returns in developing countries.

LITERATURE REVIEW

The first empirical study of APT was conducted by Brennan (1971). Brennan concluded that the two risk factors must represent a return as opposed to a single CAPM factor. However, the first published study on APT was done by Gehr (1975) as a similar version of the factor analysis approach. No further APT studies were conducted until Ross and Roll (1980) conducted their own empirical research.

Roll and Ross (1980) first checked to see if there were multiple systemic risk factors affecting the rate of return on assets as the theory suggested. The study examined 1,260 stocks that were traded on New York and the U.S. Stock Exchange between July 3, 1962, and December 31, 1972. The tests performed consist of two stages. In the first stage, the expected rates of return and element betas were estimated using the rates of return of the assets, and in the second stage, the estimated values obtained in the first stage were used to control the arbitrage pricing equation (Çelik and Kurtaran, 2016, p. 348).Gültekin, Dhrymes and Friend (1984) criticized the results of Roll and Ross in their work. In the study, they argued that APT-related tests should cover all assets available in the capital market and that not including them in the control process for any reason would lead to serious errors. In their work, they focused on various methods to check the validity of APT. In the tests they used, they examined the stability of the risk factors that explain the rates of return and whether there was any relationship between the number of financial assets included in the research and the number of elements obtained from the element analysis method. It was found that the findings obtained were different from those required by APT.

There is also a great deal of skepticism about APT's testing methods. Cheng (1996), Chen, and others (1986) emphasize the importance of the number of independent variables involved in regression. Furthermore, Cheng (1996) notes that when a researcher tests APT, one factor may be important in multivariate analysis and not again when testing in a univariate model later. A multiple collinearity between economic variables constitutes another disadvantage of this approach (Paavola, 2006).

French and Fama (1993, 1996) created a 3-factor model that captures three specific factors that influence expected return. Under the same assumptions, Zhongzhi *et al.,* (2010) proposed a new model called the Dynamic Factor Pricing Model (DFPM). In this model, it uses both old and post-old factors and combines elements of price dynamics across assets over time. Paavola (2006) has argued that it is natural for APT to perform CAPM better in a statistical sense for two reasons: APT allows for more than a single factor, and CAPM uses a single clearly defined factor.

Paavola (2006) found that the most disappointing feature of APT is that it does not identify common factors (or even numbers). APT is also not supported by the theoretical foundations of CAPM, which define the behavior of investors (Morel, 2001). Gilles and LeRoy (1990) noted that the APT does not contain useful information about prices, does not contain any clear restrictions, and can be treated as a very general asset pricing model. This generality of theoretical APT has become a major weakness for empirical APT (Koutmos and others, 1993, pp. 119-126).

Akkum and Vuran (2005) analyzed various macroeconomic factors affecting the stock returns of companies in the Turkish capital market by using multiple regression analysis method with APT. This effort was made between January 1999 and December 2002 on 20 companies that were continuously present in the Borsa İstanbul BIST30 index. In the analysis response, they found that the BIST30 index and sub-sector indices were effective in the stock returns and that APT was valid. Dhankar and Esq (2005) analyzed APT in the Indian equity market using monthly and weekly returns for the period 1991-2002. It shows that APT with multiple factors provides a better indicator of asset risk and return than CAPM, which uses beta as a single measure of risk.

DATA SET AND RESEARCH METHOD

In this study, the validity of the Arbitrage Pricing Model was tested in the stock markets of Brazil, Brasil, Bolsa, Balcão (B3 S.A) with BOVESPA as the main performance index. Five key macroeconomic factors are used in addition to the share price to perform this test: inflation, exchange rate, interest rate, industrial index, Gross Domestic Product (GDP) and stock prices. BOVESPA was used in the study.

Stock Market Index

In this study, the stock market index (BOVESPA) of B3 as the dependent factor.

Interest Rate

In this study, base interest rates received from the central bank of Brazil was used as interest rate. This is the key ratios that central banks use as tools to enforce monetary policies.

Inflation Rate

The Consumer Price Index (CPI) is a general and popular tool for measuring people's spending in an economy. In this study, CPI was used as an inflation indicator.

Gross Domestic Product

GDP is the sum of the gross added value of all established producers in the economy, as well as product taxes and subsidies that are not included in the value of finished products. It is calculated without payment for the depreciation of manufactured assets or for the depletion and deterioration of natural resources. The GDP here is used as a measure of a country's growth.

Exchange rate

The exchange rate factor is the amount of 1 US Dollar of the Brazilian Real (BRL)

Industrial Production Index

The industrial production index refers to industries belonging to sections 15-37 of the International Standard Industrial Classification (ISIC). It varies from manufacturing to recycling of products. Monthly data for this study between January 2009 and March 2020 were collected mostly from the central bank of Brazil. This period covers the period immediately after the great world economic crisis in 2008 and until the beginning of the new coronavirus outbreak in the world at the beginning of 2020. Data that were not available in monthly frequencies (high frequencies) were collected in annual frequencies (low frequencies), which were then converted into monthly frequencies. Secondary data were used in the study.

Table 1: Macroeconomic Factors Used in Analysis

Variable	Indicator	Measurement		Source	Variable Type
Stock Market Index	Index Return	$\frac{INDX_t - INDX_{t-1}}{INDX_{t-1}}$	$Log\left(\frac{INDX_t}{INDX_{t-1}}\right)$	Istanbul Stock Exchange Site	Dependent
Inflation	Consumer Price Index	$\frac{INFL_t - INFL_{t-1}}{INFL_{t-1}}$	$Log\left(\frac{INFL_t}{IINFL_{t-1}}\right)$	Central Bank of Brazil	Independent
Interest Rate	Central Bank Interest/12Month	$\frac{INTR_t}{12}$	$Log\left(\frac{INTR_t}{12}\right)$	Central Bank of Brazil	Independent
Exchange rate	TRY / US Dollar	$\frac{EXCR_t - EXCR_{t-1}}{EXCR_{t-1}}$	$Log(\frac{EXCR_t}{EXR_{t-1}})$	Central Bank of Brazil	Independent
Economic Growth	GDP	$\frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$	$Log(\frac{GDP_t}{GDP_{t-1}}$	Central Bank of Brazil	Independent
Industrial Production Index	Net Production	$\frac{PDTX_t - PDTX_{t-1}}{PDTX_{t-1}}$	$Log\left(\frac{PDTX_t}{PDTX_{t-1}}\right)$	Central Bank of Brazil	Independent

Source: Created by the author

Table 2:Abbreviation of Variables

Variable	Abbreviation
Stock Exchange Index	INDX
Inflation	INFL
Interest Rate	INTR
Exchange rate	EXCR
Economic Growth	GDP
Industrial Production Index	PDTX

Source: Created by the author

METHODOLOGY

This study uses a VAR family model (VECM) to explore the existence of a relationship between the variables. A Granger causality test was applied to determine the nature of the relationship.

The research model is determined as follows.

$$INDX_{it} = \beta_0 + \beta_1 INFL_{it} + \beta_2 INTR_{it} + \beta_3 EXCR_{it} + \beta_4 GDP_{it} + \beta_5 PDTX_{it} + \varepsilon_{it} \quad (1)$$

In Equation 1.

R_{it}: return of the stock market index
β_0: Constant
β_1: Annual change in Inflation β its sensitivity to annual change
β2: Annual change in Interest Rate
β3: Annual change in Exchange rate
β_4: Annual change in GDP
β_5: Annual change in Production Index
□ $_{it}$: Error term

First, the stationarity test of the series was performed. Next, the Cointegration test was carried out to determine if there is a long-term relationship between the variables. However, correlation doesn't necessarily mean long-term relationship; for this purpose, Johansen Cointegration Test was performed. After the cointegration test was applied, Granger Causality test was also applied to determine the relationship direction of the variables. In the Granger causality test, the series is static. Log values of the variables were used to perform the cointegration. The null hypothesis for Johansen Cointegration test states that there is no cointegration. The output from this test is based on the Trace and Maximum Eigen value statistics.

However, if there is a cointegration relationship between non-stationary series, Granger causality test is performed on Vector Error Correction Model (VECM), not on the VAR (Şentürk and Akba, 2014, p. 7). In addition, in the case of cointegration of the variables, a VECM would be made to determine the exact relationship between the variables. This model creates both short-term and long-term relationship.

The predicted VECM model is as follows:

$$\Delta INDX_t = \alpha_0 + \sum_{i=0}^{P} \alpha_1 INDX_{t-1} + \sum_{i=0}^{P} \alpha_2 INFL_{t-1} + \sum_{i=0}^{P} \alpha_3 INTR_{t-1} + \sum_{i=0}^{P} \alpha_4 EXCR_{t-1} + \sum_{i=0}^{P} \alpha_5 GDP_{t-1} + \sum_{i=0}^{P} \alpha_6 PDTX_{t-1} + \delta_1 INDX_{t-1} + \delta_2 INFL_{t-1} + \delta_3 INTR_{t-1} + \delta_4 EXCR_{t-1} + \delta_5 GDP_{t-1} + \delta_6 SÜEN_{t-1} + \varepsilon_1 \quad (2)$$

Here $\boldsymbol{\alpha}$ parameters represent short-term relationships, while $\boldsymbol{\delta}$parameters represent long-term relationships.

If the variables are cointegrated, the long-term coefficients of each variable can be estimated by an error correction model as follows. The traditional VECM regression equation for cointegrated series is as follows.

$$\Delta INDX_T = \gamma_0 + \sum_{i=0}^{p} \gamma_1 \Delta INDX_{t-1} + \sum_{i=0}^{P} \varphi_i INFL_{t-1} + \sum_{i=0}^{P} \varphi_i INTR_{t-1} + \sum_{i=0}^{P} \varphi_i EXCR_{t-1} + \sum_{i=0}^{P} \varphi_i GDP_{t-1} + \sum_{i=0}^{P} \varphi_i PDTX_{t-1} + \mu ECT_{t-1} + u_i \quad (3)$$

In the above equation (3), **γ**1 and **φ**i stand for short-term coefficients, Δ is the symbol for difference operator, **μ** is the order of delay, u_i represent the residuals and ECT_{t-1} denotes the term for error correction.

As VECM was implemented, the error term correction was introduced. In the error-correcting model, the short-term dynamics of the variables are affected by deviation from equilibrium. The model takes the difference of non-stationary variables and adds error-correction parameters between the descriptive variables to reflect the long-term adjustment to the balance. In the regression equation, it represents the delay value of the error term obtained from the cointegration equation called error correction term (Bozdağlıoğlu, 2007, p. 9). ECT (Error Correction Term) is the term for Error correction.

In this equation, ECT shows the long-term relationship between variables. The ***u*** coefficient measures the speed at which stock returns come to equilibrium after a long-term deviation. The fact that the error correction coefficient is less than 1 indicates that the system is balanced, and the fact that it is negatively marked indicates that there is a movement towards balance in case of deviation from the balance. In other words, the error correction mechanism works (Bozkurt, 2007: 166).

EMPIRICAL FINDINGS

Stationarity Test Results

Stationarity sis is a concept that refers to the fact that over time, series have a covariance due to a stationary variance and a level of delay. Time series with a stationary specificity (or no unit root) are series with a static mean and are series with variance and covariance. (Öner*et al.,* 2018:118).Most economic time series are not stationary, and this latter is obtained only at the first difference of level values or higher (Uwubanmwen and Obayagbona, 2012:10). An Augmented Dickey Fuller (ADF) test was used to analyze the presence of the unit root. In the ADF Unit Root Test, the H_0 hypothesis states that the series has a unit root, while the H_1 hypothesis states that the series is constant. The Akaike Information Criterion (AIC) Lag Length style was used to perform the test. The results are presented *at levels* and *first difference*, taking into account the intersection between variables and trends. After the series were determined to be constant at the first difference, the cointegration method developed by Johansen (1988), and then Johansen and Juselius (1990) was used to examine whether there is a long-term equilibrium relationship within the series. Before the cointegration test is applied, it is necessary to determine the lag length of the models by creating unrestricted VAR (Vector Autoregressive) in the models. Akaike Information Criterion (AIC) was generally used to determine the Lag length.

Table 3: ADF Unit Root Test Results

				LEVEL			
		INDEX	INFLATION	INTEREST RATE	GDP	PRODUCTION INDEX	EXCHANGE RATE__
Constant	t- Statistics	-8.3759	-2.3746	-0.8567	-3.7511	-2.5254	-12.1278
	Probability	0.0000	0.1514	0.7990	0.0045	0.1121	0.0000
		***	n0	n0	***	n0	***
Constant andTrend	t- Statistics	-8.3726	-2.2431	-1.5398	-3.7300	-3.1344	-12.4396
	Probability	0.0000	0.4608	0.8108	0.0242	0.1035	0.0000
		***	n0	n0	**	n0	***
				FIRST DIFFERENCE			
		d(INDEX)	d(INFLATION)	d(INTEREST)	d(GDP)	d(URETİM)	d(WHERE)
Constant	t- Statistics	-6.4906	-2.9899	-4.1791	-3.4907	-2.9020	-9.4341
	Probability	0.0000	0.0390	0.0010	0.0099	0.0482	0.0000
		***	**	***	***	**	***
ConstantandTrend	t- Statistics	-6.4421	-2.8778	-4.3649	-3.4561	-2.9987	-9.3969
	Probability	0.0000	0.1739	0.0035	0.0491	0.1371	0.0000
		***	n0	***	**	n0	***

Note: (*) represents significance of 10%, (**) represents 5%, and (***) represents significance level of 1%.

The results of the unit root test of macroeconomic variables for Brazil can be seen in Table 3. When the table is examined, it is shown that inflation rate, interest rate and industrial production index rate variables have unit roots at Constant withLevel shift. It was determined that the same variables have a unit root in constant and trend. At first difference, with 1% and 5%significant levels in constant, the variables do not have a unit root. With constant and trend, all the variables, apart from inflation rate and industrial production rate, do not have a unit root.

Findings

Time series variables can develop short and long-term relationships. When they are not static at level shifts, they can be made stationary by taking their differences. Thus, cointegration analysis allows to determine the long-term relationship between variables.The validity of the APT on the Brazilian stock exchange will depend on the relationship between its macroeconomic factors. In this section, there is an attempt to determine the relationship between the variables.

The following analysis results were obtained to determine the appropriate lag length in the first stage.

Table 4: Lag Length Detection

Lag	LogL	LR	FPE	AIC	SC	HQ
0	219.7116	ON	1.01e-09	-3.684683	-3.542256	-3.626866
1	994.5107	1456.088	2.97E-15	-16.42260	-15.42561	-16.01788
2	1087.880	165.8116*	1.11e-15*	-17.41173*	-15.56018*	-16.66011*
3	1115.894	46.85062	1.29E-15	-17.27404	-14.56792	-16.17551
4	1138.934	36.14871	1.65e-15	-17.05059	-13.48991	-15.60515

** Shows the lag length selected by the criterion.*

According to the table, LR, FPE, AIC, SIC and HQ values point all to the same lag length; and the optimal lag length was chosen as two (2).

Then, cointegration testing was performed to determine if there is a long-term relationship between the variables. The Johansen Cointegration Test was performed and the result of the cointegration test is given below.

Table 5: Johansen Cointegration Test Results

Cointegration Test by Trace Statistics				
Number of Cointegration Equations	**Eigenvalue**	**Trace Statistics**	**Critical Value**	**Probability****
Never*	0.339933	114.9245	95.75366	0.0013
Up to 1 *	0.251607	75.04473	69.81889	0.0180
Up to 2	0.177837	47.22132	47.85613	0.0573
Up to 3	0.132227	28.42288	29.79707	0.0714
Up to 4	0.097338	14.80767	15.49471	0.0633
Up to 5 *	0.050519	4.976616	3.841466	0.0257
According to the trace test, 2 cointegration equations can be established.				
Cointegration Test by Maximum Eigenvalue Statistic				
Number of Cointegration Equations	**Özdeğer**	**Maximum Eigenvalue Statistics**	**Critical value**	**Probability****
Never	0.339933	39.87975	40.07757	0.0526
Up to 1	0.251607	27.82341	33.87687	0.2218
Up to 2	0.177837	18.79845	27.58434	0.4302
Up to 3	0.132227	13.61521	21.13162	0.3974
Up to 4	0.097338	9.831050	14.26460	0.2233
Up to 5 *	0.050519	4.976616	3.841466	0.0257
The Maximum Eigenvalue test indicates that there is no cointegration at a level of 0.05.				
* Indicates that the hypothesis is rejected at a level of 0.05				
** MacKinnon-Haug-Michelis (1999) p-değerleri				

There is a discrepancy between the Trace Cointegration Test result and the Maximum Eigen value cointegration test result. According to the Trace Test, there are 2 cointegration equations at the 5% significance level.

In other words, there are cointegration equations between index price, exchange rate, inflation rate, interest rate, industrial production index and GDP. As a result, according to the Trace Cointegration Test, there is a long-term relationship between variables. This could mean that even if there are cases that may affect movement in the individual series in the short term, they will converge over time (in the long run).

The fact that there is a cointegration relationship between the variables examined shows that the tendency of the variables to deviate from the equilibrium in the short term can be handled within the framework of the vector error correction model. The existence of cointegration could indicate the presence of Granger Causality in at least one direction. A causality test was conducted to further determine the characteristics of the relationship between the variables. F-Statistics and Chi-squared from the Granger (1969) and Wald (1943) tests show short-term causal effects. Causality refers to the relationship between two events, in which one event is affected by another. In statistics, one can judge that there is causality when the value of one event or variable increases or decreases as a result of other events. The result of the Granger test for short-term causality between variables is summarized in the table below.

Table 6: Granger Causality Test Results

H0: Does not Granger cause. H1: Granger causes.	F-Statistics	Probability	Decision
Production Index Rate>> Stock Market Index	1.26592	0.2788	H0 Accept
Stock Market Index>> Production Index Rate	1.72648	0.1212	H0 Accept
Interest Rate>> Stock Market Index	0.62492	0.7098	H0 Accept
Stock Market Index>> Interest Rate	0.99259	0.4361	H0 Accept
Inflation Rate >> Stock Market Index	0.68748	0.6601	H0 Accept
Stock Market Index>> Inflation Rate	0.70129	0.6491	H0 Accept
GDP>> Stock Market Index	1.25927	0.2820	H0 Accept
Stock Market Index>> GDP	1.76445	0.1128	H0 Accept
Exchange Rate>> Stock Market Index	0.47358	0.8267	H0 Accept
Stock Market Index>> Exchange Rate	0.64307	0.6955	H0 Accept
Interest Rate >> Production Index Rate	1.16040	0.3359	H0 Accept
Production Index Rate>> Interest Rate	0.62986	0.7059	H0 Accept
Inflation Rate>> Production Index Rate	0.03031	0.9999	H0 Accept
Production Index Rate>> Inflation Rate	0.10378	0.9958	H0 Accept
GDP>> Production Index Rate	0.11553	0.9944	H0 Accept
Production Index Rate >> GDP	0.03521	0.9998	H0 Accept
Exchange Rate>> Production Index Rate	2.09236	0.0596	H0 Accept
Production Index Rate>> Exchange Rate	**3.83758**	**0.0016**	**H0 Reject**
Inflation v>> Interest Rate	1.36432	0.2392	H0 Accept
Interest Rate >> Inflation Rate	1.01028	0.4246	H0 Accept
GDP>>Interest Rate	0.76910	0.5964	H0 Accept
Interest Rate >> GDP	1.02203	0.4172	H0 Accept
Exchange Rate>> Interest Rate	1.70500	0.1305	H0 Accept
Interest Rate >> Exchange Rate	1.11592	0.3606	H0 Accept
GDP>> Inflation	0.02159	1.0000	H0 Accept
Inflation>> GDP	0.00836	1.0000	H0 Accept
Exchange Rate>> Inflation Rate	0.45226	0.8420	H0 Accept
Inflation v>> Exchange Rate	0.87430	0.5163	H0 Accept
Exchange Rate>> GDP	1.90165	0.0866	H0 Accept
GDP>> Exchange Rate	**4.19380**	**0.0008**	**H0 Reject**

According to Table 6, when the F-statistical values and the probability results are evaluated together, it is possible to conclude that there is usually a two-way causal relationship between the variables. The results shows that industrial production index rate Granger causes exchange rate but not reversely (exchange rate does not Granger cause production index rate); GDP also Granger causes exchange rate, but exchange rate does not Granger cause GDP. The decision was made at 5% significance level.

If a series of variables is found to have one or more cointegration vectors, a suitable estimation technique is a VECM, which adjusts for short-term changes and deviations in variables from equilibrium. VECM is a VAR with a boundary designed for non-stationary series between which there is cointegration. Error Correction Models (ECM) are a useful, theoretically driven approach for estimating both the short- and long-term effects of one time series on another. The term error correction is related to the fact that the deviation of the last period from the long-term equilibrium, that is, the error affects the short-term dynamics. Thus, ECMs directly predict the rate at which a dependent variable returns to equilibrium after a change in other variables.

The results of the Cointegration Equation Error Correction Term (ECT) and long-term VECM are as shown in Table 7.

Table 7: Cointegration Equation

Cointegration Equation	CointEq1
INDX (-1)	1.000000
PDTX(-1)	-1.174337
INTR (-1)	-0.129837
INFL (-1)	-0.096878
GDP (-1)	2.066698*
EXCR (-1)	1.553767***
C	-0.006355

The Error Correction Term equation (or Cointegration Equation- CE) determines the equation of long-term relationship between variables.

$$\mathrm{ECT}_{t-1} = 1.000\mathrm{INDX}_{t-1} - 1.1743\mathrm{PDTX}_{t-1} - 0.1298\mathrm{INTR}_{t-1} - 0.0969\mathrm{INFL}_{t-1} + 2.0667\mathrm{GDP}_{t-1} + 1.5538\mathrm{EXCR}_{t-1} - 0.0064 \quad (4)$$

Equation 4 shows the long-term relationship between the variables. In the long run, both GDP and the exchange rate have a positive effect on the stock market index. These are the coefficients at two significant levels in the equation.

The short-term relationship between the variables is shown by the following equation.

$$\Delta INDX_t = -0.1557ECT_{t-1} - 0.2837INDX_{t-1} + 0.0752PDTX_{t-1} + 0.368INTR_{t-1} + 0.1192INTR_{t-1} - 0.7271GDP_{t-1} + 0.2252EXCR_{t-1} + 0.0009 \quad (5)$$

Equation 5 shows a negative coefficient of the error correction term, as it is usually expected. This is the adjustment coefficient, which helps the models to return to equilibrium after deviations.

These findings demonstrate the validity of the Arbitrage Pricing Model in the Brazilian market with the macroeconomic factors selected above. In addition, diagnostic and stability tests can be used to analyze the model.

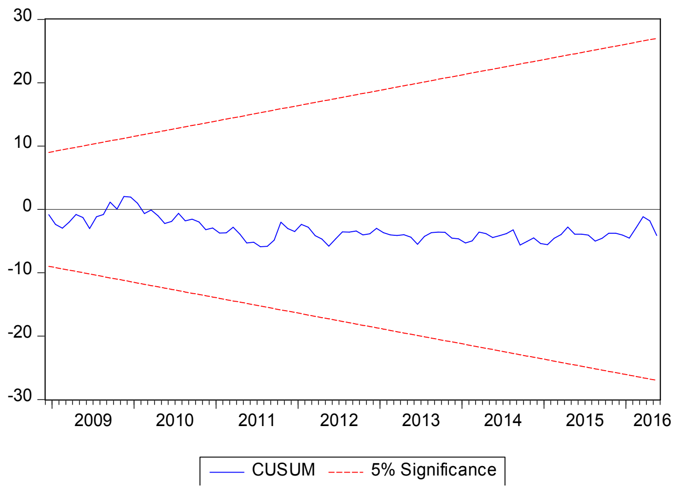
Table 8: Heteroskedasticity Test

Test of Heteroskedasticity: Breusch-Pagan-Godfrey			
F-Statistics	1.821682	**Prob. F (12,85)**	**0.0572**
Observation *R-square	20.04767	Prob. Chi-Kare (12)	0.0662
Scale description SS	18.63792	Prob. Chi-Kare (12)	0.0976

When the Breusch-Pagan-Godfrey Heteroskedasticity test results are examined in the table, the probability value is 0.0572. Therefore, since this value is greater than 5%(p=0.0572> 0.05), it can be concluded that there is no heteroskedasticitye problem.

To investigate the stability of the predicted model, the graphs of cumulative sum (CUSUM) and cumulative sum square (CUSUMSQ) (Brown et al. 1975, pp. 149-155), which use squares of the terms of reversible error and thus investigate the structural break attached to the variables, were considered. The recursive predictions of the stability test show that the model is stable. Because the curve is within the two red boundaries.

Figure 1: Recursive Predictions of Stability Test



When the above CUSUM graph is examined, it shows that there is no structural break of the variables used in the analysis, that the long-term coefficients calculated according to the VECM are stable and that the model can be predicted without using any artificial variables.

CONCLUSIONS

This study delved into the complexities of the Arbitrage Pricing Model (APM) to investigate its applicability in the Brazilian stock market, B3 S.A (formerly BOVESPA), in conjunction with various macroeconomic factors. A comprehensive analysis was conducted, covering aspects such as stationarity, cointegration, and Granger causality, to shed light on the relationship between the stock market index and macroeconomic variables, including inflation, interest rates, exchange rates, industrial production, and GDP. Models were estimated by using the data for the period of January 2009-March 2020

The findings have provided valuable insights into the dynamics of the Brazilian stock market and the role of macroeconomic factors. Firstly, it was observed that the stationarity tests confirmed the necessity of working with first differences of the variables to achieve stationarity, a common feature of economic time series. This allowed to proceed with cointegration analysis. The cointegration tests, specifically the Johansen Cointegration Test, indicated that a long-term equilibrium relationship exists between the stock market index and the selected macroeconomic variables. This suggests that, over time, these variables tend to converge and maintain a stable relationship, even though short-term fluctuations may occur.

Further, the Granger causality tests revealed interesting insights into the causal relationships between the variables. In some cases, it was found evidence of bidirectional causality, signifying that these variables have mutual influence on each other, both in the short term and long term. This interplay is particularly important for investors and policymakers seeking to understand how changes in macroeconomic factors impact the stock market and vice versa. Additionally, the VECM analysis unveiled the long-term and short-term relationships between these variables. Notably, both GDP and exchange rates exhibited significant positive effects on the stock market index in the long run, emphasizing the importance of these macroeconomic factors in shaping stock market performance. Different cointegration equations were determined with VECM and an Error Correction Term (ECT) was created. The expected result is that all ECTs are negative. After the presence both long- and short-term relationships among the variables, the resulting ECT coefficient is negative and meets the expectation: -0.1557 (0.1557%). This figure depicts the speed at which B3-BOVESPA can readjust itself after discrepancies occurred in the market.

In summary, this study contributes to the understanding of the Brazilian stock market's relationship with macroeconomic variables within the framework of the Arbitrage Pricing Model. While APM is a complex and multifaceted model, our research has demonstrated its relevance in explaining the dynamics of stock returns in the Brazilian context. Investors and policymakers can utilize these insights to make informed decisions, manage risk, and navigate the intricacies of the Brazilian financial landscape. Additionally, our study underscores the significance of considering both short-term and long-term relationships when analyzing the impact of macroeconomic factors on stock market behavior, providing a valuable perspective for future research and market participants.

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