International Journal of Innovation Scientific Research and Review
Vol. 05, Issue, 11, pp.5512-5522, November 2023
Available online at http://www.journalijisr.com
SJIF Impact Factor 2023: 6.599

IJISRR
ISSN: 2582-6131

APPLICATION OF DOUBLE ABOODH-SUMUDU DECOMPOSITION METHOD TO SOLVE NONLINEAR SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS

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Received 20th September 2023; Accepted 25th October 2023; Published online 30th November 2023

ABSTRACT

In this work, we develop a new method to obtain approximate solutions of nonlinear coupled partial di erential equations with the help of double Aboodh-Sumudu decomposition method (DASDM). The nonlinear term can easily be handled with the help of Adomian polynomials. The results of the present technique have closed agreement with approximate solutions obtained with the help of Adomian decomposition method (ADM).

Keywords:. Double Aboodh-Sumudu decomposition method, Adomian decom-position method, nonlinear Partial di erential equations.

INTRODUCTION

The topic of partial di erential equations is one of the most important subjects in mathematics and other sciences. Therefore it is very important to know meth-ods to solve such partial di erential equations. Two of the most popular methods for solving partial di erential equations are the integral transforms method and Adomian decomposition method (ADM)[5, 10]. In the literature, there are many di erent types of integral transforms are introduced and applied to nd the solu-tion of linear partial di erential equations such as Laplace transform [7], Sumudu transform [4], Aboodh transform [9] and so on. The decomposition method has been shown to solve e ciently and accurately a large class of linear and nonlinear ordinary, partial, deterministic or stochastic di erential equations [6, 11, 12, 13]. The method is very well suited to physical problems since it does not require un-necessary linearization, perturbation, discretization, or any unrealistic assump-tions. The ADM is relatively easy to implement, and it can be used with other methods. It can also be used to solve both initial value problems and boundary value problems. In [5], the authors showed that the intimate connection between the Sumudu transform theory and decomposition method was demonstrated in the solution of nonlinear partial di erential equations.

The main objective of this paper is to obtained the exact solutions of coupled nonlinear partial di erential equations with initial value problems by using dou-ble Aboodh-Sumudu transform algorithm based on decomposition method.

First, we recall the de nitions of Aboodh, Sumudu and double Aboodh-Sumudu transforms.

The Aboodh transform of the real function h(y) of exponential order is de ned over the set of functions

$$\mathcal{M} = \left\{ h(y) : \exists K, \tau_1, \tau_2 > 0, |h(y)| < Ke^{|y|\tau_i},\ y \in (-1)^i \times [0, \infty),\ i = 1, 2 \right\},$$

Key words and phrases. Double Aboodh-Sumudu decomposition method, Adomian decom-position method, nonlinear Partial differential equations.

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by the following integral

$$A[h(y)] = H(q) = \frac{1}{q}\int_0^\infty e^{-qy}h(y)dy, \ \ \tau_1 \le q \le \tau_2.$$

And the inverse Aboodh transform is

$$A^{-1}[H(q)] = h(y) = \frac{1}{2\pi i}\int_{\omega-i\infty}^{\omega+i\infty} qe^{qy}H(q)dq, \ \ \omega \ge 0.$$

For further details and properties of the Aboodh transform and its derivatives we refer to [8, 9].

The Sumudu transform of the function $h(t)$ is defined over the set of functions

$$\mathcal{N} = \Big\{h(t) : \exists M, \rho_1, \rho_2 > 0, |h(t)| < Me^{\frac{|t|}{\rho_j}}, \ t \in (-1)^j \times [0,\infty), \ j = 1, 2\Big\},$$

by

$$S[h(t)] = H(r) = \frac{1}{r}\int_0^\infty e^{-\frac{t}{r}}h(t)dt.$$

Moreover, the inverse of Sumudu transform is

$$S^{-1}[H(r)] = h(t) = \frac{1}{2\pi i}\int_{\omega-i\infty}^{\omega+i\infty} \frac{1}{r}e^{\frac{t}{r}}H(r)dr, \ \ \omega \ge 0.$$

For further details and properties of the Sumudu transform and its derivatives we refer to [3, 4].

The double Aboodh-Sumudu transform of the continuous function $h(y,t), \ \ y, t > 0$ is denoted by the operator $A_yS_t[h(y,t)] = H(q,r)$ and defined by

$$\begin{aligned} A_yS_t[h(y,t)] &= H(q,r) = \frac{1}{qr}\int_0^\infty\int_0^\infty e^{-(qy+\frac{t}{r})}h(y,t)dydt \\ &= \frac{1}{qr}\lim_{\alpha\to\infty,\beta\to\infty}\int_0^\alpha\int_0^\beta e^{-(qy+\frac{t}{r})}h(y,t)dydt. \end{aligned}$$

It converges if the limit of the integral exists, and diverges if not.

And the inverse double Aboodh-Sumudu transform is defined by

$$h(y,t) = A_y^{-1}S_t^{-1}[H(q,r)] = \frac{1}{(2\pi i)^2}\int_{\gamma_1-i\infty}^{\gamma_1+i\infty} qe^{qy}dq\Bigg\{\int_{\gamma_2-i\infty}^{\gamma_2+i\infty}\frac{1}{r}e^{\frac{t}{r}}H(q,r)dr\Bigg\},$$

where γ_1 and γ_2 are real constants.

Double Aboodh-Sumudu transform for second partial derivatives property

$$\begin{aligned} A_yS_t\Big[\frac{\partial^2 h(y,t)}{\partial y^2}\Big] &= q^2H(q,r) - S[h(0,t)] - \frac{1}{q}S[h_y(0,t)], \\ A_yS_t\Big[\frac{\partial^2 h(y,t)}{\partial t^2}\Big] &= \frac{1}{r^2}H(q,r) - \frac{1}{r^2}A[h(y,0)] - \frac{1}{r}A[h_t(y,0)], \\ A_yS_t\Big[\frac{\partial^2 h(y,t)}{\partial y\partial t}\Big] &= \frac{q}{r}H(q,r) - \frac{q}{r}A[h(y,0)] - \frac{1}{q}S[h_t(0,t)], \end{aligned}$$

where $A[.]$ and $S[.]$ denote to single Aboodh transform and single Sumudu transform respectively.

In [2], some fundamental properties of the double Aboodh-Sumudu transform and its derivatives were established. Moreover, double Aboodh-Sumudu transform for some functions are showed.
We consider the general inhomogeneous nonlinear partial differential equation with initial conditions given below:

$$Lu(x,y,t) + Ru(x,y,t) + Nu(x,y,t) = f(x,y,t) \tag{1.1}$$

$$u(x,y,0) = g_1(x,y), \qquad u_t(x,y,0) = g_2(x,y), \tag{1.2}$$

where $L = \frac{\partial^2}{\partial t^2}$ is the second order derivative which is assumed to be easily invertible, R is the remaining linear differential operator, Nu represents the nonlinear terms and $f(x,y,t)$, $g_1(x,y)$ and $g_2(x,y)$ are known functions.
The methodology consists of applying double Aboodh-Sumudu transform first on both sides of Eq. (1.1)

$$A_yS_t[Lu(x,y,t)] + A_yS_t[Ru(x,y,t)] + A_yS_t[Nu(x,y,t)] = A_yS_t[f(x,y,t)]. \tag{1.3}$$

Using the differentiation property of double Aboodh-Sumudu transform, we have

$$\begin{aligned} \frac{1}{r^2}u(x,q,r) &- \frac{1}{r^2}A[u(x,y,0)] - \frac{1}{r}A[u_t(x,y,0)] + A_yS_t[Ru(x,y,t)] \\ &+ A_yS_t[Nu(x,y,t)] = A_yS_t[f(x,y,t)], \end{aligned} \tag{1.4}$$

Using given initial conditions and arrangement, Eq. (1.4) becomes

$$\begin{aligned} u(x,q,r) &= A[g_1(x,y)] + rA[g_2(x,y)] + r^2A_yS_t[f(x,y,t)] \\ &- r^2A_yS_t[Ru(x,y,t)] - r^2A_yS_t[Nu(x,y,t)]. \end{aligned} \tag{1.5}$$

Application of inverse double Aboodh-Sumudu transform to (1.5) leads to

$$\begin{aligned} u(x,y,t) &= A_y^{-1}S_t^{-1}\Big[A[g_1(x,y)] + rA[g_2(x,y)] + r^2A_yS_t\big[f(x,y,t)\big]\Big] \\ &- A_y^{-1}S_t^{-1}\Big[r^2A_yS_t\big[Ru(x,y,t)\big] + r^2A_yS_t\big[Nu(x,y,t)\big]\Big]. \end{aligned} \tag{1.6}$$

The second step in double Aboodh-Sumudu decomposition method is that we represent solution as an infinite series:

$$u(x,y,t) = \sum_{i=0}^{\infty} u_i(x,y,t), \tag{1.7}$$

and the nonlinear term can be decomposed as

$$Nu(x,y,t) = \sum_{i=0}^{\infty} A_i, \tag{1.8}$$

where A_i are Adomian polynomials [12] of $u_0, u_1, u_2, ..., u_n$ and it can be calculated by formula

$$A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[N\sum_{i=0}^{\infty}\lambda^i u_i\right]_{\lambda=0}. \tag{1.9}$$

Substituting Eq. (1.7) and Eq. (1.8) in Eq. (1.6), we get

$$\sum_{i=0}^{\infty} u_i(x,y,t) = A_y^{-1}S_t^{-1}\Big[A[g_1(x,y)] + rA[g_2(x,y)] + r^2A_yS_t[f(x,y,t)]\Big]$$
$$- A_y^{-1}S_t^{-1}\Big[r^2A_yS_t\Big[R\sum_{i=0}^{\infty} u_i(x,y,t)\Big] + r^2A_yS_t\Big[\sum_{i=0}^{\infty} A_i\Big]\Big]. \quad (1.10)$$

On comparing both sides of the Eq. (1.10) and by using standard Adomian decomposition method (ADM), we then define the recurrence relations as

$$u_0(x,y,t) = A_y^{-1}S_t^{-1}\Big[A[g_1(x,y)] + rA[g_2(x,y)] + r^2A_yS_t[f(x,y,t)]\Big], \quad (1.11)$$

$$u_1(x,y,t) = -A_y^{-1}S_t^{-1}\Big[r^2A_yS_t[Ru_0(x,y,t)] + r^2A_yS_t[A_0]\Big], \quad (1.12)$$

$$u_2(x,y,t) = -A_y^{-1}S_t^{-1}\Big[r^2A_yS_t[Ru_1(x,y,t)] + r^2A_yS_t[A_1]\Big]. \quad (1.13)$$

In more general, the recursive relation is given by

$$u_{i+1}(x,y,t) = -A_y^{-1}S_t^{-1}\Big[r^2A_yS_t[Ru_i(x,y,t)] + r^2A_yS_t[A_i]\Big], \ i \geq 0. \quad (1.14)$$

The recurrence relation generates the solution of (1.1) in series form given by

$$u(x,y,t) = u_0(x,y,t) + u_1(x,y,t) + u_2(x,y,t) + ... + u_i(x,y,t) + ... \quad (1.15)$$

2. Applications

In order to illustrate the applicability and efficiency of the double Aboodh-Sumudu decomposition method, we apply this method to solve some examples.

Example 2.1. *Consider the following nonlinear partial differential equation*

$$u_{tt}(x,y,t) + u^2(x,y,t) - u_x^2(x,y,t) = 0, \qquad t > 0, \quad (2.1)$$

subject to the initial conditions

$$u(x,y,0) = 0, \qquad u_t(x,y,0) = e^{x+y}. \quad (2.2)$$

Applying double Aboodh-Sumudu transform algorithm, we get

$$\frac{1}{r^2}u(x,q,r) - \frac{1}{r^2}A[u(x,y,0)] - \frac{1}{r}A[u_t(x,y,0)] = A_yS_t[u_x^2(x,y,t) - u^2(x,y,t)].$$

Rearranging the terms and using given initial conditions, we have

$$u(x,q,r) = \frac{r}{q(q-1)}e^x + r^2A_yS_t\big[u_x^2(x,y,t) - u^2(x,y,t)\big]. \quad (2.3)$$

By applying the inverse double Aboodh-Sumudu transform for Eq. (2.3), we get

$$u(x,y,t) = te^{x+y} + A_y^{-1}S_t^{-1}\Big[r^2A_yS_t\big[u_x^2(x,y,t) - u^2(x,y,t)\big]\Big]. \quad (2.4)$$

The double Aboodh-Sumudu decomposition method assumes a series solution of the function $u(x,y,t)$ is given by

$$u(x,y,t)=\sum_{i=0}^{\infty}u_i(x,y,t). \tag{2.5}$$

Using Eq. (2.5) into Eq. (2.4) we get

$$\sum_{i=0}^{\infty}u_i(x,y,t)=te^{x+y}+A_y^{-1}S_t^{-1}\left[r^2A_yS_t\left[\sum_{i=0}^{\infty}A_i(u)-\sum_{i=0}^{\infty}B_i(u)\right]\right], \tag{2.6}$$

where A_i and B_i are Adomian polynomials that represents nonlinear terms. So Adomian polynomials are given as follows:

$$\sum_{i=0}^{\infty}A_i(u)=u_x^2(x,y,t), \quad \sum_{i=0}^{\infty}B_i(u)=u^2(x,y,t). \tag{2.7}$$

The few components of the Adomian polynomials are given as follow:

$$A_0(u)=u_{0x}^2, \ \ A_1(u)=2u_{0x}u_{1x}, \ \ \ldots, \ \ A_i(u)=\sum_{r=0}^{i}u_{rx}u_{(i-r)x}, \tag{2.8}$$

$$B_0(u)=u_0^2, \ \ B_1(u)=2u_0u_1, \ \ \ldots, \ \ B_i(u)=\sum_{r=0}^{i}u_ru_{i-r}. \tag{2.9}$$

From Eqs. (2.6) and (2.7) we obtain

$$u_0 = te^{x+y}, \tag{2.10}$$

$$\sum_{i=0}^{\infty}u_{i+1}(x,y,t) = A_y^{-1}S_t^{-1}\left[r^2A_yS_t\left[\sum_{i=0}^{\infty}A_i(u)-\sum_{i=0}^{\infty}B_i(u)\right]\right], \ i\geq 0 \tag{2.11}$$

Then the first few components of $u_i(x,y,t)$ follows immediately upon setting

$$\begin{aligned} u_1(x,y,t) &= A_y^{-1}S_t^{-1}\left[r^2A_yS_t[A_0(u)-B_0(u)]\right] \\ &= A_y^{-1}S_t^{-1}\left[r^2A_yS_t[u_{0x}^2-u_0^2]\right] \\ &= A_y^{-1}S_t^{-1}\left[r^2A_yS_t[t^2e^{2x+2y}-t^2e^{2x+2y}]\right] \\ &= A_y^{-1}S_t^{-1}[r^2A_yS_t[0]]=0. \end{aligned} \tag{2.12}$$

Similarly, $u_2(x,y,t)=u_3(x,y,t)=u_4(x,y,t)=0$ and so on. Thus, we obtain the solution of (2.1) by double Aboodh-Sumudu decomposition method as

$$u(x,y,t)=\sum_{i=0}^{\infty}u_i(x,y,t)=te^{x+y}. \tag{2.13}$$

Which is the required solution.

Example 2.2. *Consider the system of nonlinear partial differential equations*

$$\begin{aligned} u_t+vu_y+u=1, \\ v_t-uv_y-v=1, \end{aligned} \tag{2.14}$$

with initial conditions

$$\begin{array}{l} u(y,0) = e^{y}, \\ v(y,0) = e^{-y}. \end{array} \tag{2.15}$$

Applying the double Aboodh-Sumudu transform to both sides of equations (2.14), we have

$$\begin{array}{l} \frac{1}{r}U(q,r) - \frac{1}{r}A[u(y,0)] = A_y S_t[1] - A_y S_t[vu_y + u], \\ \frac{1}{r}V(q,r) - \frac{1}{r}A[v(y,0)] = A_y S_t[1] + A_y S_t[uv_y + v]. \end{array} \tag{2.16}$$

Application of single Aboodh transform to (2.15) and substitute in (2.16), we have

$$\begin{array}{l} U(q,r) = \frac{1}{q(q-1)} + \frac{r}{q^2} - rA_y S_t[vu_y + u], \\ V(q,r) = \frac{1}{q(q+1)} + \frac{r}{q^2} + rA_y S_t[uv_y + v]. \end{array} \tag{2.17}$$

By taking the inverse double Aboodh-Sumudu transform in (2.17), our required recursive relation is given by

$$\begin{array}{l} u(y,t) = e^{y} + t - A_y^{-1} S_t^{-1}\Big[rA_y S_t\big[vu_y + u\big]\Big], \\ v(y,t) = e^{-y} + t + A_y^{-1} S_t^{-1}\Big[rA_y S_t\big[uv_y + v\big]\Big]. \end{array} \tag{2.18}$$

The recursive relations are

$$\begin{array}{rcl} u_0(y,t) &=& e^{y}, \\ u_{i+1}(y,t) &=& t - A_y^{-1} S_t^{-1}\left[rA_y S_t\left[\sum_{i=0}^{\infty} C_i(v,u) + \sum_{i=0}^{\infty} u_i\right]\right], \ i \geq 0 \\ v_0(y,t) &=& e^{-y}, \\ v_{i+1}(y,t) &=& t + A_y^{-1} S_t^{-1}\left[rA_y S_t\left[\sum_{i=0}^{\infty} D_i(u,v) + \sum_{i=0}^{\infty} v_i\right]\right], \ i \geq 0, \end{array} \tag{2.19}$$

where $u(y,t)$ and $v(y,t)$ are linear terms represented by the decomposition series and $C_i(v,u)$ and $D_i(u,v)$ are Adomian polynomials representing the nonlinear terms [12]. The few components of Adomian polynomials are given as follow

$$
\begin{aligned}
C_0(v,u) &= v_0u_{0y},\\
C_1(v,u) &= v_0u_{1y} + v_1u_{0y},\\
C_2(v,u) &= v_0u_{2y} + v_1u_{1y} + v_2u_{0y},\\
C_3(v,u) &= v_0u_{3y} + v_1u_{2y} + v_2u_{1y} + v_3u_{0y},\\
&\vdots\\
C_i(v,u) &= \sum_{n=0}^{i} v_nu_{(i-n)y},\\
D_0(u,v) &= u_0v_{0y},\\
D_1(u,v) &= u_0v_{1y} + u_1v_{0y},\\
D_2(u,v) &= u_0v_{2y} + u_1v_{1y} + u_2v_{0y},\\
D_3(u,v) &= u_0v_{3y} + u_1v_{2y} + u_2v_{1y} + u_3v_{0y},\\
&\vdots\\
D_i(u,v) &= \sum_{n=0}^{i} u_nv_{(i-n)y}.
\end{aligned}
$$

Using the derived Adomian polynomials into (2.19), we obtain

$$
\begin{aligned}
u_0(y,t) &= e^y,\\
v_0(y,t) &= e^{-y},\\
u_1(y,t) &= t - A_y^{-1}S_t^{-1}\big[rA_yS_t[C_0(v,u)+u_0]\big] = t - A_y^{-1}S_t^{-1}\big[rA_yS_t[v_0u_{0y}+u_0]\big]\\
&= t - A_y^{-1}S_t^{-1}\big[rA_yS_t[1+e^y]\big] = t - A_y^{-1}S_t^{-1}\big[\frac{r}{q^2} + \frac{r}{q(q-1)}\big]\\
&= -te^y,\\
v_1(y,t) &= t + A_y^{-1}S_t^{-1}\big[rA_yS_t[D_0(u,v)+v_0]\big] = t + A_y^{-1}S_t^{-1}\big[rA_yS_t[u_0v_{0y}+v_0]\big]\\
&= t + A_y^{-1}S_t^{-1}\big[rA_yS_t[-1+e^{-y}]\big] = t + A_y^{-1}S_t^{-1}\big[-\frac{r}{q^2} + \frac{r}{q(q+1)}\big]\\
&= te^{-y},\\
u_2(y,t) &= -A_y^{-1}S_t^{-1}\big[rA_yS_t[C_1(v,u)+u_1]\big] = -A_y^{-1}S_t^{-1}\big[rA_yS_t[v_0u_{1y}+v_1u_{0y}+u_1]\big]\\
&= -A_y^{-1}S_t^{-1}\big[rA_yS_t[-te^y]\big] = -A_y^{-1}S_t^{-1}\big[-\frac{r^2}{q(q-1)}\big]\\
&= \frac{t^2}{2!}e^y,\\
v_2(y,t) &= A_y^{-1}S_t^{-1}\big[rA_yS_t[D_1(u,v)+v_0]\big] = A_y^{-1}S_t^{-1}\big[rA_yS_t[u_0v_{1y}+u_1v_{0y}+v_1]\big]\\
&= A_y^{-1}S_t^{-1}\big[rA_yS_t[te^{-y}]\big] = A_y^{-1}S_t^{-1}\big[\frac{r^2}{q(q+1)}\big]\\
&= \frac{t^2}{2!}e^{-y}.
\end{aligned}
$$

In the same way we can get

$$\begin{aligned} u_3(y,t) &= -\frac{t^3}{3!}e^y, \\ v_3(y,t) &= \frac{t^3}{3!}e^{-y}, \end{aligned}$$

and so on for other components. Therefore, the series solutions obtained by double Aboodh-Sumudu decomposition method are given by

$$\begin{aligned} u(y,t) &= \sum_{i=0}^{\infty} u_i(y,t) = e^y\Big(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + ...\Big) = e^{y-t}, \\ v(y,t) &= \sum_{i=0}^{\infty} v_i(y,t) = e^{-y}\Big(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...\Big) = e^{-y+t}. \end{aligned}$$

Which is same as solution obtained by Sumudu decomposition method [5].

Example 2.3. *Consider the system of nonlinear partial differential equations*

$$\begin{aligned} u_y(x,y,t) &- v_x w_t = -1, \\ v_y(x,y,t) &- w_x u_t = 1, \\ w_y(x,y,t) &- u_x v_t = -5, \end{aligned} \tag{2.20}$$

with initial conditions

$$\begin{aligned} u(x,0,t) &= x + 3t, \\ v(x,0,t) &= x + 3t, \\ w(x,0,t) &= -x + 3t. \end{aligned} \tag{2.21}$$

Taking the double Aboodh-Sumudu transform to both sides of equations (2.20), we have

$$\begin{aligned} qu(x,q,r) - \frac{1}{q}S[u(x,0,t)] &= -\frac{1}{q^2} + A_y S_t[v_x w_t], \\ qv(x,q,r) - \frac{1}{q}S[v(x,0,t)] &= \frac{1}{q^2} + A_y S_t[w_x u_t], \\ qw(x,q,r) - \frac{1}{q}S[w(x,0,t)] &= -\frac{5}{q^2} + A_y S_t[u_x v_t]. \end{aligned} \tag{2.22}$$

Application of single Sumudu transform to (2.21) then substitute in (2.22) and rearranging the terms, we have

$$\begin{aligned} u(x,q,r) &= \frac{1}{q^2}(x+3r) - \frac{1}{q^3} + \frac{1}{q}A_y S_t[v_x w_t], \\ v(x,q,r) &= \frac{1}{q^2}(x+3r) + \frac{1}{q^3} + \frac{1}{q}A_y S_t[w_x u_t], \\ w(x,q,r) &= \frac{1}{q^2}(-x+3r) - \frac{5}{q^3} + \frac{1}{q}A_y S_t[u_x v_t]. \end{aligned} \tag{2.23}$$

By taking the inverse double Aboodh-Sumudu transform in (2.23), we get

$$\begin{aligned} u(x,y,t) &= x+3t-y+A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[v_xw_t\right]\right], \\ v(x,y,t) &= x+3t+y+A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[w_xu_t\right]\right], \\ w(x,y,t) &= -x+3t-5y+A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[u_xv_t\right]\right]. \end{aligned} \tag{2.24}$$

The recursive relations are

$$\begin{aligned} u_0(x,y,t) &= x-y+3t, \\ u_{i+1}(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[\sum_{i=0}^{\infty}E_i(v,w)\right]\right],\ i\geq 0. \\ v_0(x,y,t) &= x+y+3t, \\ v_{i+1}(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[\sum_{i=0}^{\infty}F_i(w,u)\right]\right],\ i\geq 0, \\ w_0(x,y,t) &= -x-5y+3t, \\ w_{i+1}(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t\left[\sum_{i=0}^{\infty}G_i(u,v)\right]\right],\ i\geq 0, \end{aligned} \tag{2.25}$$

where $E_i(v,w)$, $F_i(w,u)$, and $G_i(u,v)$ are Adomian polynomials representing the nonlinear terms [12] in above equations. The few components of Adomian polynomials are given as follow

$$\begin{aligned} E_0(v,w) &= v_{0x}w_{0t}, \\ E_1(v,w) &= v_{1x}w_{0t}+v_{0x}w_{1t}, \\ &\vdots \\ E_i(v,w) &= \sum_{n=0}^{i}v_{nx}w_{(i-n)t} \\ F_0(w,u) &= w_{0x}u_{0t}, \\ F_1(w,u) &= w_{1x}u_{0t}+w_{0x}u_{1t}, \\ &\vdots \\ F_i(w,u) &= \sum_{n=0}^{i}w_{nx}u_{(i-n)t} \\ G_0(u,v) &= u_{0x}v_{0t}, \\ G_1(u,v) &= u_{1x}v_{0t}+u_{0x}v_{1t}, \\ &\vdots \\ G_i(u,v) &= \sum_{n=0}^{i}u_{nx}v_{(i-n)t} \end{aligned}$$

In view of this recursive relations we obtained other components of the solution as follows

$$
\begin{aligned}
u_1(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[E_0(v,w)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[v_{0x}w_{0t}]\right] = A_y^{-1}S_t^{-1}\left[\frac{3}{q^3}\right] = 3y,\\
v_1(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[F_0(w,u)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[w_{0x}u_{0t}]\right] = A_y^{-1}S_t^{-1}\left[\frac{-3}{q^3}\right] = -3y,\\
w_1(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[G_0(u,v)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[u_{0x}v_{0t}]\right] = A_y^{-1}S_t^{-1}\left[\frac{3}{q^3}\right] = 3y,\\
u_2(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[E_1(v,w)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[v_{1x}w_{0t}+v_{0x}w_{1t}]\right] = 0,\\
v_2(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[F_1(w,u)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[w_{1x}u_{0t}+w_{0x}u_{1t}]\right] = 0,\\
w_2(x,y,t) &= A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[G_1(u,v)]\right] = A_y^{-1}S_t^{-1}\left[\frac{1}{q}A_yS_t[u_{1x}v_{0t}+u_{0x}v_{1t}]\right] = 0.
\end{aligned}
$$

Similarly, $u_3(x,y,t) = v_3(x,y,t) = w_3(x,y,t) = 0$ and so on for rest terms. Therefore, the solution of system (2.20) are given below

$$
\begin{aligned}
u(x,y,t) &= \sum_{i=0}^{\infty} u_i(x,y,t) = x + 2y + 3t,\\
v(x,y,t) &= \sum_{i=0}^{\infty} v_i(x,y,t) = x - 2y + 3t,\\
w(x,y,t) &= \sum_{i=0}^{\infty} w_i(x,y,t) = -x - 2y + 3t.
\end{aligned}
$$

3. Conclusion

In the present paper, double Aboodh-Sumudu transform method combined with Adomian decomposition method which so-called the double Aboodh-Sumudu decomposition method (DASDM) is applied to solve nonlinear coupled partial differential equations with initial conditions. Three examples have been presented. The results of these examples tell us that both methods can be used alternatively for the solution of high-order initial value problems.

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