

Research Article

MODELING OF THE PROCESSES ASSOCIATED WITH THE OIL EXTRACTION SYSTEM THROUGH DEPTH PUMPING

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ABSTRACT

Aims: A method for predicting the behavior of the crude oil extraction system by deep pumping is presented. Predicting the behavior of the rod pumping system requires solutions obtained through differential equations to which a whole set of boundary conditions is applied. For pumping rod assemblies, the vibrating string equation and boundary conditions describing initial rod assembly pressure and velocity, polished rod motion, and pump operating conditions are used.

Methodology: The classical pumping system is described by a fairly flexible mathematical model based on numerically solved differential equations. Surface dynamogram and bottom (pump) dynamogram plus intermediate dynamograms can be determined for a wide range of bottom and surface conditions. This technique that will be presented allows the simulation of vast operating conditions, and the data obtained through these methods are useful in the design and use of rod pumping systems.

Keywords: Oil, Deep extraction, modelling.

INTRODUCTION

The simulation of the behavior of the rod assembly is done with the help of the one-dimensional vibrating string equation, using the Damping factor and the boundary conditions of the rod - pump system, [1],[2],[3],[4].

$$\frac{\partial^2}{\partial t^2} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\pi a v}{2L} \frac{\partial u(x,t)}{\partial t} \quad (1)$$

This equation describes longitudinal vibrations in rods and is therefore ideal in rod-pump applications.

Also, in this mathematical model is incorporated the shock reflection phenomenon along the gasket, an important characteristic of real systems. The Damping factor used in equation (1) eloquently modifies the obtained solutions, even if the effects of friction (independent of viscosity) of tubing rods and the hysteresis of the steel from which the rods are made are also present.

Fortunately, the viscosity-independent effects are relatively small, so the approximation of the Damping factor in the vibrating string equation is adequate, [5]. In equation 1, the Damping factor is represented by the dimensionless coefficient v which is determined according to the conditions in the field; this coefficient has a narrow range in which it varies (approx. 0 +0.2), [6].

Due to mathematical conventions, in equation 1 the gravitational acceleration is omitted. The effect of gravity on the loading and elongation of the pump rod assembly will be treated separately.

The movement of the polished rod is a function of the geometry of the pumping unit, the motor torque and the number of beats per minute achieved by it, [7].

Important boundary conditions will be used to determine the movement of the polished rod.

From geometric considerations, it can be demonstrated that the position of the polished rod depending on the angle of the crank, θ (fig. 1.) is given by the relationship [8]:

$$u(u, \theta) = L_3 \left[\sin^{-1} \left(\frac{L_1 \sin \theta}{h} \right) + \cos^{-1} \left(\frac{h^2 + L_3^2 + L_1^2}{2L_3 h} \right) \right] \quad (2)$$

$$h = \sqrt{L_1^2 + L_2^2 + 2L_1 L_2 \cos \theta} \quad (3)$$

This equation, obtained from the general solution of the "four-bar linkage" problem, can describe the kinematics of any pumping unit.

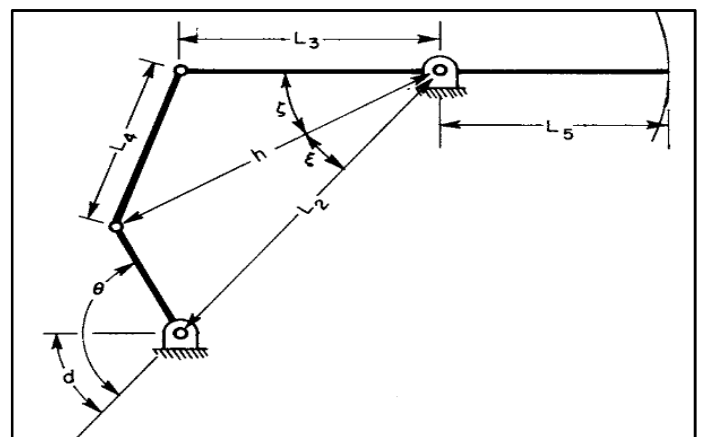


Fig. 1. Scheme of the pumping unit with rocker arm "four-bar linkage",

If the rate of variation of the motor is negligible, the angular velocity of the crank is constant and equation 2 can be used to determine the position of the polished rod as a function of time.

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Angular speed with which the motor operates is determined by its own torque-speed characteristics and the imposed moment.

The moment at the motor (with which it operates) is given by the net moment from the loading of the polished rod and from the opposite moment from the counterbalancing effect. The moment given by the load in the rod is obtained as a product of the load at the rod and the moment factor (torque), [9].

The torque factor, from mechanics, is given by, [10], [11]:

$$TF = \frac{L_1 L_2 L_3 \sin(\zeta - \theta - \psi)}{L_3} \tau = \cos^{-1} \left(\frac{h^2 + L_3^2 - L_1^2}{2L_3 h} \right) \quad (4)$$

$$\psi = \sin^{-1} \left(\frac{L_3 \sin \tau}{L_1} \right) \quad (5)$$

The counterbalancing moment that opposes the moment produced by the load in the probe is given by the relation:

$$CBT = L_3 W_f \cos(\theta - d) \quad (6)$$

where "d" is the phase angle required to orient the counterbalancing effect in relation to the weight.

Thus, the moment imposed on the engine is the algebraic sum of the moment in the probe and the counterbalancing moment:

$$NT = TF(PRL) + CBT \quad (7)$$

This net torque, together with the motor's torque-speed characteristics, determines the instantaneous speed at which the motor operates. Thus the instantaneous angular velocity of the crank can be determined depending on the transmission ratio of the reducer and the size of the washer, to reflect the variation of the engine speed as a response to the variation of the net torque. Inertia effects were not considered in this study.

The most important boundary condition in deep rod pumping is that which describes the operation of the bottom pump.

Undoubtedly, the mathematical model according to which the bottom pump worked, raised the biggest problems in the analysis of the operation of the extraction system with pumping rods. In this field, many studies and researches have formulated explicit expressions, which describe the behavior of the rod seal and the pump, but unfortunately these expressions have proven inadequate in relation to the real situations in the probe. It is therefore improbable that a classical analysis of the pump-pump system will provide a picture as close as possible to the one in the field.

For this reason, it was agreed to write the following relationship describing the operation of the bottom pump:

$$\alpha u(L, t) + \beta \frac{\partial u(L, t)}{\partial x} = P(t) \quad (8)$$

where the parameters a,b and P(t) depend on the type of pump operations that will be simulated.

Example:

a) $\alpha = 0, \beta = 1, P(t) = 0$ transform equation (7) into the form:

$$\frac{\partial u(L, t)}{\partial x} = 0$$

which means that the pump works without the load, situation in the probe when the working valve remains open.

b) $\alpha = 1, \beta = 0, P(t) = uc$ transform equation (7) into the form:

$$u(L, t) = uc$$

which means that the pump remains fixed in a certain position uc. This situation can be found when the fluid load is transferred from the rods to the tubing or from the tubing to the rods.

c) $\alpha = 1, \beta = 1, P(t) = Wf/EA$ transform equation (7) into the form:

$$EA \frac{\partial u(L, t)}{\partial x} = W_f$$

that is, a constant load Wf is applied to the pump, a situation encountered when the fluid is lifted to the surface.

It is instructive to show the options of parameters α, β and P(t) in the case of gas circulating through the pump. In this case, the pump diagram has the form of fig. 2.

- t1 - the moment when the working valve closes
- t2 - the time when the fixed valve opens
- t3 - the time when the fixed valve closes
- t4 - the moment when the working valve opens

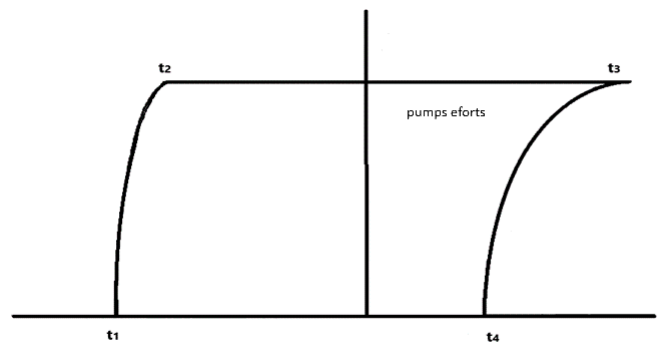


Fig. 2. Typical graph for the working valve at certain times Parameters α, β and P(t) will be:

$$\alpha = 0$$

$$\beta = 1 \quad t1 \leq t \leq t2$$

$$P(t) = G_1 [u(L, t_1) - u(L, t)]$$

Where t1, is the time when the working valve closes and a new pumping cycle starts again.

Functions G1 and G2 determine the shape of the diagram when the fluid load is transferred from rods to tubing or vice versa.

These functions depend on the pressure-volume relations of the mixture that begins to be pumped. If the amount of gas passing through the pump decreases, the volumetric efficiency increases and the pump dynamogram tends to a rectangular shape.

A suitable choice for α, β and P(t) can also simulate this situation.

The mathematical model described above is quite complicated and an analytical solution can be obtained with difficulty. It is much easier and more efficient to obtain solutions using partial differential equations.

The equation of the vibrating string becomes: (written using finite differences)

$$x U^{\tau+1} = \frac{x^{+1} U^{\tau+} + x^{-1} U^{\tau-} - x U^{\tau-1+} + x U^{\tau}}{1+r} \quad (9)$$

Where :

$$\begin{aligned} \tau &= 0, 1, 2, \dots \\ x &= 0, 1, 2, \dots, x^* \\ x^* U^{\tau+1} &= u(x, t + \Delta t) \\ x^* U^\tau &= u(x, t) \\ x^* U^{\tau-1} &= u(x, t - \Delta t) \\ x^{*+1} U^\tau &= u(x + \Delta x, t) \\ x^{*-1} U^\tau &= u(x - \Delta x, t) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 u(x, t)}{\partial t^2} &\approx \frac{u(x, t + \Delta t) - 2u(x, t) + u(x, t - \Delta t)}{\Delta t^2} \\ \frac{\partial^2 u(x, t)}{\partial x^2} &\approx \frac{u(x + \Delta x, t) - 2u(x, t) + u(x - \Delta x, t)}{\Delta x^2} \\ \frac{\partial u(x, t)}{\partial t} &\approx \frac{u(x, t + \Delta t) - u(x, t)}{\Delta t} \end{aligned}$$

Making these notations, the solutions of equation (9), for a Damping factor equal to zero, satisfy the equation of the vibrating string regardless of the value of Δx . If the Damping factor is also considered, the solutions above are no longer exact, but they are still close. Experimentally, it was found that for a small Damping factor, the errors introduced by this numerical method are lower than 0.5%. Using digital computers, this method allows obtaining quick and economical solutions.

In particular, the bottom pump conditions become, [12]:

$$x^* U^\tau = \frac{\Delta x P^\tau + 2\beta x^{*-1} U^\tau - 0.5\beta x^{*-1} U^\tau}{\alpha \Delta x + \frac{3}{2\beta}} \quad (10)$$

Where $-x^* U^\tau$ reflect the displacement of the pump.

Equation (10) is obtained directly from equation (6) by replacing the derivative with a difference. The mechanism of writing the pump conditions in differential form is the same as in equation (6).

An appropriate choice of parameters a,b and P(t) is necessary depending on the operation of the valves. The times the valves are open and closed are determined by the computer through the following tests:

T1

$$\text{As long as: } \frac{3}{2} \cdot z^* U^\tau - z^* U^\tau - 2 \cdot x^{*-1} U^\tau + \frac{1}{2} \cdot x^{*-2} U^\tau = 0 \text{ (there is no load on the pump).}$$

The computer notices (records) when the difference $x^* U^\tau - x^* U^{\tau-1}$ changes the sign (from positive value to negative value). This means that the pump has reached its lowest position, at which point the work valve has closed.

T2:

$$\text{As long as: } \frac{3}{2} \cdot x^* U^\tau - 2 \cdot x^{*-1} U^\tau + \frac{1}{2} \cdot x^{*-2} U^\tau > 0 \text{ (load on the pump),}$$

The computer notices (records) when:

$$\frac{EA}{\Delta x} \left[\frac{3}{2} \cdot x^* U^\tau - 2 \cdot x^{*-2} U^\tau + \frac{1}{2} \cdot x^{*-2} U^\tau \right] = W_f$$

At this point the fluid load is completely taken over by the rods and the fixed valve is open.

T3:

$$\text{As long as: } \frac{EA}{\Delta x} \left[\frac{3}{2} \cdot x^* U^\tau - 2 \cdot x^{*-2} U^\tau + \frac{1}{2} \cdot x^{*-2} U^\tau \right] = W_f, \text{ The computer notices (records) when } x^* U^\tau - x^* U^{\tau-1} \text{ and changes sign (from negative value to positive value). At this moment the pump has reached its highest position and the fixed valve closes.}$$

T4:

As long as: $\frac{3}{2} \cdot x^* U^\tau - 2 \cdot x^{*-1} U^\tau + \frac{1}{2} \cdot x^{*-2} U^\tau > 0$ (load on the pump), the computer determines when the above expression becomes zero. At this moment the fluid load is taken over by the tubing and the working valve opens.

The dynamic loads on the polished rod can be calculated using the differential version of Hooke's law.

$${}^0 F^\tau = \frac{EA}{\Delta x} \left[-\frac{3}{2} {}^0 U^\tau + 2 {}^1 U^\tau - \frac{1}{2} {}^2 U^\tau \right]$$

The equation of the vibrating string is written without the gravitational term, which must be treated separately.

Thus, the total load on the polished rod (PRL), static and dynamic, is given by:

$$PRL = {}^0 F^\tau + W_b \quad (11)$$

where W_b is the weight of the rods in the fluid. Similarly, the load and position of the pump can be correlated taking into account the effect of gravity. The dynamic load of the pump is given by the differential equation:

$$x^* F^\tau = \frac{EA}{\Delta x} \left[-\frac{3}{2} x^* U^\tau + 2 x^{*-1} U^\tau + \frac{1}{2} x^{*-2} U^\tau \right]$$

and the actual value of the load is obtained by subtracting the force due to buoyancy from the dynamic load:

$$PL = x^* F^\tau - F_b$$

The effects of gravity on the position of the pump are taken into account by adding the static elongation of the rods:

$$x^* Z^\tau = x^* U^\tau + \text{static elongation}$$

CONCLUSIONS

The article presents a method for predicting the behavior of a deep pumping crude oil extraction system using numerically solved differential equations. These simulations allow the analysis of various operating conditions and are useful in the design and use of rod pumping systems. Also discussed are pump rod liner simulation, polished rod motion, and bottom pump operation.

The final conclusion provides the conclusions of the study, emphasizing the importance of periodic monitoring of rod-pumped wells and the use of dynamograms to ensure efficient operation and prevent mechanical problems. Best practices are recommended for optimizing the performance of pumping systems and directions for future research are suggested, including optimization of operational parameters and implementation of advanced technologies for equipment monitoring and maintenance.

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