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A TWO-STEP NONLINEAR EXPLICIT THIRD-ORDER METHOD FOR INITIAL VALUE PROBLEMS (IVPS)

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ABSTRACT

This work presents a two-step nonlinear explicit third-order method for solving Initial Value Problems (IVPs) whose solutions possess singularities. The qualitative properties - local truncation, stability and convergence of the constructed method are also discussed.

Keywords: Ordinary Differential Equations, Two-Step Method, Initial Value Problems, Nonlinear, Singularities. 2000 MSC: 65L05, 65L06, 65L20.

INTRODUCTION

Many numerical methods for solving

$$y' = f(x, y(x)), x \in [x_0, X], y(x_0) = \eta \tag{1}$$

are based on the assumption that the solution is locally represent able by a polynomial. However, when 1 or its theoretical solution y(x) is known to possess a singularity, it becomes inappropriate to represent the solution in the neighborhood of the singularity by a polynomial [1], [2]. The solutions produced around singularity points by Runge-Kutta type methods, Obrechk off and general linear multistep methods are usually very poor as these methods are based on local representation by polynomials [4], [1], [5], [3]. [4] is a pioneer work on quadrature formulas based on rational interpolating functions. Seen to be effective in the neighborhood of the singularity and even beyond are the rational interpolation schemes proposed in [5] and [2]. In [2], the author replaced the general rational function of [4] with $F(x)=\frac{P_m(x)}{Q_n(x)}$ where $P_m(x)$ and $Q_n(x)$ are respectively polynomial of degree *m* and *n*. The limitation of classical methods in solving problems with singularities can be overcome by constructing methods that use a rational functions as local representation of the theoretical solution [1]. This is true as rational functions are more appropriate for the representation of functions close to singularities than polynomials. Based on this approach, several methods have been proposed [6], [7], [8], [9] [10].The works [6],[7], [9], [10], [11], [12], [14] established that solution around singularity point are well approximated by this approach. In this work, a two-step nonlinear explicit third-order method for solving (1) is presented. The starting values are obtained using the explicit one-step method proposed in [14]. The local truncation error and absolute stability of the method are also discussed.

CONSTRUCTION OF PROPOSED METHOD

Here, we present the construction of the proposed two-step nonlinear explicit third-order method for solving (1). This section assumes that the theoretical solution y(x) can be locally represented by the rational interplant

$$r(x) = \frac{a_0 + a_1 x + a_2 x^2}{b_0 + x} \tag{2}$$

satisfying the following:

$$\left.\begin{aligned} r(x_{n+j}) &= y_{n+j}, j = 0, 1, 2 \\ r^{(i)}(x_{n+j}) &= y^{(i)}_{n+j}, j = 0, i = 1, 2 \end{aligned}\right\} \tag{3}$$

Substituting for expressions and simplifying (3) yields

$$y_n = \frac{a_2 x^2 + a_1 x + a_0}{b_0 + x_n} \tag{4}$$

$$y_{n+1} = \frac{a_2 (h + x_n)^2 + a_1 (h + x_n) + a_0}{b_0 + h + x_n} \tag{5}$$

$$y_{n+2} = \frac{a_2 (2h + x_n)^2 + a_1 (2h + x_n) + a_0}{b_0 + 2h + x_n} \tag{6}$$

$$y'_n = \frac{a_2 x_n (2b_0 + x_n)^2 + a_1 b_0 - a_0}{(b_0 + x_n)^2} \tag{7}$$

$$y'_{n+1} = \frac{a_2 (h + x_n)(2b_0 + h + x_n) + a_1 b_0 - a_0}{(b_0 + h + x_n)^2} \tag{8}$$

Eliminating the undetermined coefficients a_0, a_1, a_2 and b_0 in (4) results in

$$y_{n+2} = \frac{2hy_{n+1}y'_n + hy'_{n+1}(2hy'_n + y_n) - y_n^2 + 5y_{n+1}y_n - 4y_{n+1}^2}{h(2y'_n + y'_{n+1}) + 2y_n - 3y_{n+1}} \tag{9}$$

The resulting scheme (9) is a two-step, nonlinear, explicit method. We shall refer to (9) as **TSNEM** which is the method proposed in this work.

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Local truncation error and absolute stability of constructed method

In this section, we consider the associated local truncation error (lte) and the absolute stability properties of the proposed method.

Local Truncation Error

Local Truncation Error: The local truncation error T_{n+k} at $x_n + k$ is defined as

$$T_{n+k} = y(x_n + kh) - y_{n+k} \tag{10}$$

where, $y(x_n)$ is the theoretical solution. From the above, the local truncation error of the proposed **TSNEM** method is obtained as

$$T_{n+2} = \frac{1}{2}h^4 y^{(4)}(x) + \frac{2h^4 y^{(3)}(x)^2}{9y''(x)} y_{n+k} \tag{11}$$

Order of an Ordinary Differential Equation

Order of the proposed **TSNEM** method: A numerical method is said to be of order p if p is the largest integer for which

$T_{n+k} = 0(h^{p+1})$ for every n and $p \geq 1$. Using the above, the order of the proposed **TSNEM** method is obtained as p = 3.

Conclusion

In this work, a two-step nonlinear explicit third-order method for solving Initial Value Problems (IVPs) whose solutions possess singularities had been derived. The qualitative properties - local truncation, stability and convergence of the constructed method was also discussed. The method is therefore recommended for problems whose solution possesses singularity.

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